AA 284a Advanced Rocket Propulsion

Lecture 10
Hybrid Rocket Propulsion
Design Issues

Prepared by
Arif Karabeyoglu

Department of Aeronautics and Astronautics
Stanford University

May 14, 2012
Axial Flow Hybrid Rocket

- For an axial flow hybrid the oxidizer flows parallel to the axis of the propulsion system through a cylindrical cavity(s) inside fuel grain
- The cross sectional shape of the cylindrical opening (port) can be
  - Circle, Triangle, D-shape or any other complex form
- There could be multiple cylindrical openings (ports), multi-port hybrid
  - Two ports: Double-D
  - Four ports: Quad
  - Five ports: Quad+1
  - Larger number of ports utilizes single or double row wagon wheel configuration
  - AMROC motor :15+1 ports
- Fuel utilization (minimizing the sliver fraction) dictates the shapes of the ports once the number of ports and overall port configuration is selected
- The fuel sliver fraction increases with the increasing number of corners
- Note that the hydraulic diameters of the ports must be matched for even burning
- Axial variation of the port geometry is also possible
- Most simple port design is a single circular geometry. This is the most efficient shape for fuel utilization. No corners
- We will limit the discussions to single port hybrids
Axial Flow Hybrid Rocket – Axial Variation of Regression Rate

- The local instantaneous regression rate expression:

$$\dot{r}(x, t) = a \ G^n x^m$$

- Note that classical theory predicts $n=0.8$ and $m=-0.2$

- First consider the axial variation of regression rate for a given instant in time. For simplicity assume that the port shape and hydraulic diameter is independent of $x$.

- The axial mass balance in the fuel port yields ($C_p$ is the circumference)

$$\dot{m}(x) = \dot{m}_{ox} + \int_0^x \rho_f C_p \dot{r}(x') \ dx'$$

- Convert to flux ($A_p$ is the port area) and substitute the regression rate expression

$$G(x) = G_{ox} + \frac{a \ \rho_f C_p}{A_p} \int_0^x G^n x'^m \ dx'$$
Axial Flow Hybrid Rocket – Axial Variation of Regression Rate

- Convert the integral equation to a differential equation

\[
\frac{dG}{dx} = a \rho_f \frac{C_p}{A_p} G^n x^m
\]

- Integrate by the separation of variables

\[
G(x) = \left( G_{ox} + a \rho_f \left( \frac{1-n}{1+m} \right) \frac{C_p}{A_p} x^{1+m} \right)^{1/1-n}
\]

- Note that

\[
G_{ox} = \frac{\dot{m}_{ox}}{A_p}
\]

- For circular port

\[
\frac{C_p}{A_p} = \frac{4}{D_p}
\]

- To obtain regression rate substitute the flux expression into the regression rate equation
Axial Flow Hybrid Rocket – Axial Variation of Regression Rate

- Constant port area assumption implies that the derived formula for the axial variation of the regression rate is only valid at t=0
- The regression rate variation can be more than 20%
- Axial change in port diameter more than 10% is rarely observed
- This due to the self-correcting behavior: As one part of the port opens up more, the local flux decreases, resulting in a decrease in the regression rate
- Typically space averaged port diameter is used in the calculations
As discussed in the previous section, the axial variation of the port diameter and the regression rate will be ignored. Introduce the space averaged port diameter and the regression rate.

Use the simplified space averaged regression rate expression which is assumed to be valid at any instant of the hybrid operation:

\[ \dot{r}(t) = a \, G_{ox}^n \]

Using the definition of the regression rate, the dynamic equation for the space averaged port diameter can be written as:

\[ \frac{dD_p}{dt} = 2 \, \dot{r} = 2 \, a \, G_{ox}^n \]

For a circular port hybrid:

\[ \frac{dD_p}{dt} = 2^{2n+1} a \, \frac{\dot{m}_{ox}^n}{\pi^n \, D_p^{2n}} \]
For constant oxidizer flow rate, this expression can be integrated to obtain the port diameter as a function of time

\[ D_p(t) = \left[ D_{pi}^{2n+1} + \frac{(2n+1)2^{2n+1}a}{\pi^n} \dot{m}_{ox}^n t \right]^{1/(2n+1)} \]

The instantaneous flux, regression rate, fuel mass flow rate, O/F can be calculated from

\[ G_{ox}(t) = \frac{4\dot{m}_{ox}}{\pi D_p^2} \quad \dot{r}(t) = aG_{ox}^{n} \quad \dot{m}_f(t) = \rho_f \pi D_p L \dot{r} \quad \frac{O}{F} = \frac{\dot{m}_{ox}}{\dot{m}_f} \]

Based on O/F, total mass flow rate and nozzle geometry, one can determine the chamber pressure and thrust as a function of time

\[ P(t) = \frac{\left(\dot{m}_{ox} + \dot{m}_f\right) c_{theo} \eta_c}{A_{nt} C_d} \quad T(t) = \left(\dot{m}_{ox} + \dot{m}_f\right) c_{theo} \eta_c C_{Ftheo} \eta_n \]

Numerical integration is required for complex oxidizer flow rate schedules
Axial Flow Hybrid Rocket – Design Example

- Port Diameter, cm
- Time, sec

- Thrust, kg
- Time, sec

- Regression Rate, mm/sec
- Time, sec

- Oxidizer Mass Flux, kg/m^2 sec
- Time, sec

Keywords: Axial Flow Hybrid Rocket, Design Example, Regression Rate, Port Diameter, Thrust, Oxidizer Mass Flux, Time.
AA 284a Advanced Rocket Propulsion

Axial Flow Hybrid Rocket – Design Example

- **O/F Shift**: O/F in a hybrid rocket changes in time
  - Shift due to port opening
  - Shift due to throttling

- Thrust change is due to
  - Change in the fuel mass flow rate
  - O/F shift
  - Chamber pressure drop
Axial Flow Hybrid Rocket – Scaling Laws

• For constant oxidizer flow rate, the following formula relates the initial and final port diameters

\[
\left( \frac{D_{pf}}{D_{pi}} \right)^{2n+1} - 1 = \frac{(2n+1)2^{2n+1}a}{D_{pi}^{2n+1} \pi^n} \dot{m}_{ox} t_b
\]

• The final port diameter can be solved in terms of the total oxidizer mass \( M_{ox} \)

\[
D_{pf} = \left[ \frac{(2n+1) \pi^n a}{\pi^n} \frac{M_{ox}^{n(1-n)} t_b}{1 - \left( D_{pi}/D_{pf} \right)^{2n+1}} \right]^{\frac{1}{2n+1}}
\]

• The fuel grain length can be related to the O/F

\[
L = \frac{4M_{ox}}{\pi \rho_f (O/F) \left( D_{pf}^2 - D_{pi}^2 \right)}
\]
AA 284a Advanced Rocket Propulsion

Axial Flow Hybrid Rocket – Design Process

- Select
  - Propellants
  - Port diameter ratio
    - Structural design constraint
    - Bore stress increases with increasing diameter ratio
    - A typical value is 2
    - Note that the diameter ratio is related to the volumetric loading

\[ VL = 1 - \left( \frac{D_{pi}}{D_{pf}} \right)^2 \]

- The diameter ratio also determines the flux ratio during the burn
  - \( O/F \) (from optimal Isp)
  - Burn time
    - from optimal trajectory and constraints
    - Minimize the gravity loss under the acceleration and Qmax constraints

- Propellant mass (from mission requirement)
- Chamber pressure
- Nozzle area ratio

- Use the design equations to determine the geometrical parameters: grain dimensions, nozzle throat and exit areas
Axial Flow Hybrid Rocket – Design Process

• The grain geometry is critical in achieving an efficient packing in a hybrid rocket system

• For liquid systems packing is relatively easy since the tank geometries can be selected freely

• For solids packing is less of an issue
  – Solids are denser
  – A wide range of fuel grain geometries are possible (fuel generation rate is proportional to the fuel surface area)

• For most applications small $L/D$ values are desirable. Maximize the grain diameter (Written in terms of the total impulse ($I_{tot}$), volumetric loading ($VL$) and burn time ($t_b$))

$$D_{pf} = \left[ \left( \frac{2n+1}{2^n} \right)^{2n+1} a \left( \frac{O/F}{1+O/F} \right)^n I_{tot} t_b^{1-n} \right] \frac{1}{2n+1}$$

• Note that grain length is

$$L = \frac{4}{\pi \rho_f VL} \frac{I_{tot}}{I_{sp g_o}} \frac{1}{(1+O/F)D_{pf}^2}$$
**AA 284a Advanced Rocket Propulsion**

*Axial Flow Hybrid Rocket – Scaling Trends*

- Grain Diameter increases with increasing regression rate coefficient
- Grain L/D decreases with increasing regression rate coefficient

- Grain Diameter increases with increasing impulse
- Grain L/D slightly increases with increasing total impulse
Axial Flow Hybrid Rocket – Scaling Trends

- Grain Diameter increases with increasing burn time (for $n < 0.5$)
- Grain L/D decreases with increasing burn time

For slow burning fuels, L/D for a single port system is unacceptably large

This is the driving force for multi-port designs
The oxidizer to fuel ratio of a liquid rocket is directly controlled by adjusting the oxidizer and fuel mass flow rates.

The O/F of a solid system is constant (or passively programmed) since fuel and oxidizer are premixed in the solid phase.

The O/F of a hybrid rocket shifts during the operation since the fuel generation is determined by the physics and chemistry of the combustion process.

The O/F for a single circular hybrid can be written as:

\[
\frac{O}{F} = \frac{\dot{m}_{ox}}{\dot{m}_f} = \frac{\pi^n D_p^{2n} \dot{m}_{ox}}{\rho_f \pi D_p \alpha 4^n \dot{m}_{ox}^{1-n}} = \frac{D_p^{2n-1} \dot{m}_{ox}^{1-n}}{\rho_f \pi^{1-n} \alpha 4^n \rho_f L}
\]

For typical hybrids the port exponent 2n-1 and the oxidizer mass flow rate exponent 1-n are both positive (0.5<n<0.8)

Thus O/F increases with time (diameter effect) for constant oxidizer mass flow rate. For Dpf/Dpi=2 and n=0.62, O/F shifts by a factor of 1.18. Effect of this kind of a shift on the c* efficiency is small due to the flat nature of the c* curve around the optimum O/F.

Also O/F increases with increasing oxidizer mass flow rate (for n=0.62, a throttling ratio of 10:1 changes the O/F by a factor of 2.4)

If n=0.5 no shift due to port opening, if n=1 no shift due to throttling.

Oxidizer flow rate can be programmed to keep the O/F constant \( \dot{m}_{ox} \propto D_p^{(2n-1)/(1-n)} \)

In a classical hybrid both O/F and thrust can not be controlled simultaneously.

Aft oxidizer injection allows one to schedule the required thrust profile at a constant O/F.
AA 284a Advanced Rocket Propulsion

Fuel Grain Stress Distribution-Pressure Loading

- Stress Distribution
- Pressure Loading
- Single Circular Port
- Paraffin-based Fuel

Material approaches failure boundary as b/a increases

Failure Boundaries:

\[
\tau = \frac{\sigma_r - \sigma_\theta}{2} < \tau_y \quad \sigma = \max(\sigma_r, \sigma_\theta) < \sigma_f
\]
Hybrid Rocket Fuel Port Designs

- Single Circular Port
- Double-D Port Design
- Cruciform Port
- 4+1 Port Design
- 6+1 Port Wagon Wheel

Decreasing Regression Rate
Regression Rate Data for Various Hybrid Propellants

- **HTPB/LOX:**
  \[ \dot{r} = 3.043 \times 10^{-2} G_{ox}^{0.681} \]

- **HTPB/Escorez/LOX**
  \[ \dot{r} = 2.061 \times 10^{-2} G_{ox}^{0.68} \]

- **HDPE/LOX**
  \[ \dot{r} = 2.340 \times 10^{-2} G_{ox}^{0.62} \]

- **Paraffin/LOX**
  \[ \dot{r} = 11.70 \times 10^{-2} G_{ox}^{0.62} \]

- **Paraffin/N2O**
  \[ \dot{r} = 15.50 \times 10^{-2} G_{ox}^{0.50} \]

(Units are mm/sec and kg/m²·sec)
• Pre-combustion chamber: vaporization of the oxidizer (2:1 ellipse)
• Post-combustion chamber: mixing and reaction of the unburned fuel and oxidizer. Increase effective $L^*$ (hemisphere)